

Math-3 Class

12/10/2023 Review



Math Class Plan

- **2nd Grade Math Basic Review**

- **Place Value**

- **Digits, Counting Blocks, Regrouping**
- **Ones, Tens, and Hundreds**

- **Comparing**

- **The number Line**
- **Greater or Less**

- **Addition**

- **Number Line**
- **Adding and Taking Aways**
- **Doubles**

- **Subtraction**

- **Breaking**
- **Parentheses**

- **Advanced 2nd Grade Math**

- **Expressions**

- **Evaluating**
- **Missing Number**
- **Symbols**

- **Big Numbers**

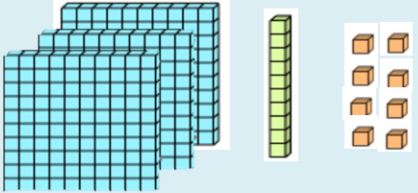
- **Larger Place Value**
- **Addition & Subtraction (option)**

- **Algorithms**

- **Step-by-Step**
- **Stacking**

- **Multiplication Table**

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms



Ones, Tens, and Hundreds

<i>Hundreds</i>	<i>tens</i>	<i>ones</i>
<u>3</u>	<u>1</u>	<u>8</u>
3	1	8

3 is the hundreds digits
1 is tens digits
8 is ones

318



Math Lesson Review

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms

BIG $>$ <small>small</small>	=	<small>small</small> $<$ BIG
Is greater than	Is equal to	Is Less than



Order from greatest to least and reverse

87, 11, 203, 96, 451, 8, 112

Greatest to Least: 451 > 203 > 112 > 96 > 87 > 11 > 8

Least to Greatest: 8 < 11 < 87 < 96 < 112 < 203 < 451



Math Lesson Review

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms

Traditional Way:

1. Visual
2. Number Line
3. Place Value
4. Add and Take Away
5. Stack Up

$$\begin{array}{r} 25 \longrightarrow \text{first addend} \\ + 32 \longrightarrow \text{second addend} \\ \hline 57 \longrightarrow \text{sum} \end{array}$$



Math Lesson Review

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms

Traditional Way:

1. Visual

2. Number Line

3. Place Value

4. Breaking and Take Away

5. Partitioning use Parentheses ()

6. Stack Up

$$\begin{array}{r}
 16 \longrightarrow \text{minuend} \\
 - 10 \longrightarrow \text{subtrahend} \\
 \hline
 06 \longrightarrow \text{difference}
 \end{array}$$



Math Lesson Review

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms

An **expression** uses numbers and operations like $+$ and $-$ to stand for a value.

Evaluating – evaluating an expression is to find its value.

$+$ and $-$, operated from **left to right**.
Evaluate **inside parentheses first**.

$$\begin{aligned}
 & 59 + (30 - 17) + 13 = ? \\
 & = 59 + \underbrace{13} + 13 \\
 & = 72 + 13 \\
 & = 85
 \end{aligned}$$

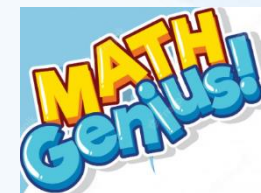


Math Lesson Review

- Place Value
- Comparing
- Addition
- Subtraction
- Expressions
- Big Numbers
- Algorithms

hundred millions	ten millions	millions	hundred thousands	tens thousands	thousands	hundreds	tens	ones	Read in English		
1	7	4	,	8	7	0	,	9	3	2	one hundred seventy-four million, eight hundred seventy thousand, nine hundred thirty-one
9	0	0	,	0	9	0	,	0	0	0	nine hundred million and ninty thousand
	1	2	,	3	4	5	,	0	6	0	Twelve million, three hundred forty-five thousand, sixty
				3	4	5	,	9	8	6	Three hundred forty-five thousand, nine hundred eighty-six



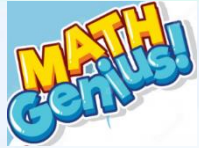


Read the Big Numbers

We learned three digits number earlier. Now, let's see the number has more digits.

Read in English

hundred millions	ten millions	millions	,	hundred thousands	tens thousands	thousands	,	hundreds	tens	ones	
1	7	4	,	8	7	0	,	9	3	2	one hundred seventy-four million, eight hundred seventy thousand, nine hundred thirty-one
9	0	0	,	0	9	0	,	0	0	0	nine hundred million and ninty thousand
	1	2	,	3	4	5	,	0	6	0	Twelve million, three hundred forty-five thousand, sixty
				3	4	5	,	9	8	6	Three hundred forty-five thousand, nine hundred eighty-six
	7	8	,	3	4	0	,	0	0	0	Seventy-eight million, three hundred forty thousand.
7	0	0	,	0	2	4	,	6	3	8	Seven hundred million, twenty-four thousand, six hundred thirty-eight



Complementary Numbers

a pair of numbers, which is obtained by subtracting a number from base.

Base = 10:

Number	Comp.
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1

Base = 100:

Number	Comp.
10	90
20	80
30	70
40	60
50	50
66	34
75	25
25	75
35	65

Base = 1000:

Number	Comp.
100	900
200	800
300	700
400	600
500	500
660	340
750	250
250	750
350	650

Base 1000: what is the complementary number of 555?

445





Tricks in Addition

Practice 6

$$9997+4+99+998+3+9$$

$$9997+4+99+998+3+9$$

$$=9997+3+1+99+998+2+1+9$$

$$=10000+100+1000+10$$

$$=11110$$



Practice 6

$$9997+4+99+998+3+9$$

$$9997+4+99+998+3+9$$

$$=9997+3+1+99+998+2+1+9$$

$$=10000+100+1000+10$$

$$=11110$$



Practice -7

$$\begin{aligned} & 29999+2999+299+29 \\ & =30000-1+3000-1+300-1+30-1 \\ & =33330-4 \\ & =33326 \end{aligned}$$





Tricks in Subtraction

From left to right, higher place value to lower place value

Practice -1

$$241 - 150 =$$

$$\begin{aligned} & 241 - 150 = \\ &= 200 + 41 - 100 - 50 \\ &= 100 + 41 - 50 \\ &= 50 + 41 \\ &= 91 \end{aligned}$$



3 x 4...
x 2... + 2...
x 3 =



Addition and Subtraction

From left to right. Parentheses first.

$$67-58-3= \quad 60-50+7-8-3=10-4=6$$

$$21+35+28= \quad 20+30+20+1+5+8=70+14=84$$

$$88-43-8= \quad 80-40+8-8-3=40-3=37$$



3 x 4...
x 2... + 2...
x 3 =



Addition and Subtraction

Rules for Parentheses

$$A-B-C = A-(B+C)$$

$$A-(B-C) = A-B+C$$

$$A-(B+C) = A-B-C$$

$$\begin{aligned} 900-384-116 &= \\ &= 900-(384+116) \\ &= 900-500=400 \end{aligned}$$

$$\begin{aligned} 657-(157+180) &= \\ &= 657-157-180 \\ &= 500-180 \\ &= 320 \end{aligned}$$



Introduce to Multiplication

What is multiplication? Multiplying means repeated addition of a number. (The number must all be the same before we can use it to multiply.)

Example 1: How many roses are in total in the picture below?

3 group, each group has 5 roses

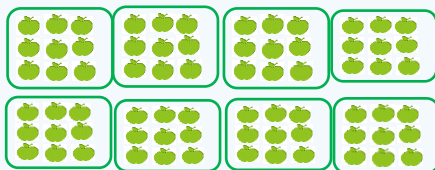


Group
↑

$$5 + 5 + 5 = 15 \quad \longrightarrow \quad 5 \times 3 = 15$$

Example 3: How many Apples are in total in the picture below?

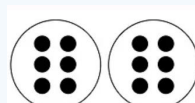
6 group, each group has 9 Apples.



$$9 + 9 + 9 + 9 + 9 + 9 = 54$$

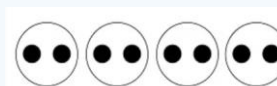
Same as $9 \times 6 = 54$

Example 2: How many black dots are in total in the pictures below?



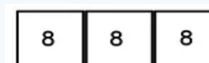
2 groups of 6
or $6 + 6$
or 2×6

$$= 12$$



4 groups of 2
or $2 + 2 + 2 + 2$
or 4×2

$$= 8$$

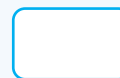


3 groups of 8
 $8 + 8 + 8$
 3×8

$$= 24$$

Example 4: How many Apples are in total in the picture below?

3 group, each group has 0 Apples.



$$0 + 0 + 0 = 0$$

Same as $0 \times 3 = 0$

Example 4: How many Apples are in total in the picture below?

3 group, each group has 1 Apples.



$$1 + 1 + 1 = 3$$

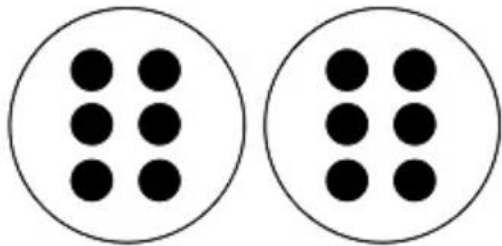
Same as $1 \times 3 = 3$



Introduce to Multiplication

What is multiplication? **Multiplying means repeated addition of a number.**
(The number must all be the same before we can use it to multiply.)

Example 2: How many black dots are in total in the pictures below?



2 groups of 6
or $6 + 6$
or 2×6

Using Addition:

$$6 + 6 = 12$$

Using Multiplication:

$$6 \times 2 = 12$$

Groups



Example 1:

$$7 \times 9 = 63$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline 63 \end{array}$$

$$12 \times 9 = 108$$

$$\begin{array}{r} 12 \\ \times 9 \\ \hline 108 \end{array}$$

Diagram illustrating the multiplication process for 12 x 9. A dashed green line connects the 1 in the tens place of 12 to the 9 in the ones place of the multiplier, with a '1' above it, indicating the carry. A dashed red box encloses the 2 in the ones place of 12 and the 9 in the ones place of the multiplier, indicating the first multiplication step (2 x 9 = 18).

Long Multiplication:

$$\begin{array}{r} 123 \\ \times 5 \\ \hline 615 \end{array}$$

$$\begin{array}{r} 12 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$



X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144



Please Excuse My Dear Aunt Sally

P Parentheses $()$

E Exponents 8^2

M Multiplication \times

D Division \div

A Addition $+$

S Subtraction $-$

$$\begin{aligned} & 6 \times 2 + 5 \times (3+4) \\ & = 6 \times 2 + 5 \times 7 \\ & = 12 + 35 \\ & = 47 \end{aligned}$$

Example 5:

$$\begin{aligned}(16 + 14) \times (17 - 15) - 20 + 2 \\&= 30 \times 2 - 20 + 2 \\&= 60 - 20 + 2 \\&= 40 + 2 \\&= 42\end{aligned}$$



Trick for multiply by 11

$$7 \times 11 = 77$$

$$11 \times 11 = 121$$

$$11 \times 12 = 132 \quad (3=1+2)$$

$$11 \times 25 = 275$$

$$11 \times 77 = 847$$

$$11 \times 99 = 1089$$

$$11 \times 123 = 1353$$

$(5 = 2+3 \quad 3=1+2)$

$$11 \times 999 = 10989$$



Example 6:

$$292+307+295+303+296+305$$

$$=300-8+300+7+300-5+300+3+300-4+300+5$$

$$=300*6-8+7-5+3-4+5$$

$$=1800-2$$

$$=1798$$



3. $AB+BA$

$$\begin{aligned} & AB+BA \\ &=AA+BB \\ &=10A+A+10B+B \\ &=11A+11B \\ &=11(A+B) \end{aligned}$$



Example 7:

$$\begin{aligned}63+36 \\ &=11*(6+3) \\ &=11*9 \\ &=99\end{aligned}$$

$$\begin{aligned}56+65 \\ &=11*(5+6) \\ &=11*11 \\ &=121\end{aligned}$$



3. AB-BA

$$AB-BA$$

$$=10A+B-10B-A$$

$$=9A-9B$$

$$=9*(A-B)$$



Example 8:

$$91-19$$

$$=9 \times (9-1)$$

$$=9 \times 8$$

$$=72$$

$$82-28$$

$$=9 \times (8-2)$$

$$=9 \times 6$$

$$=54$$



Review trick by multiply 10, 100, 1000, 10000:

$$5 \times 1 = 5$$

$$5 \times 10 = 50$$

$$5 \times 100 = 500$$

$$5 \times 1000 = 5,000$$

$$5 \times 10000 = 50,000$$



Example 2:

$$1. \quad 9603 \times 3 =$$

X	9000	600	0	3
3	27000	1800	0	9

$$= 27000 + 1800 + 0 + 9 = 28809$$

Answers: 28809

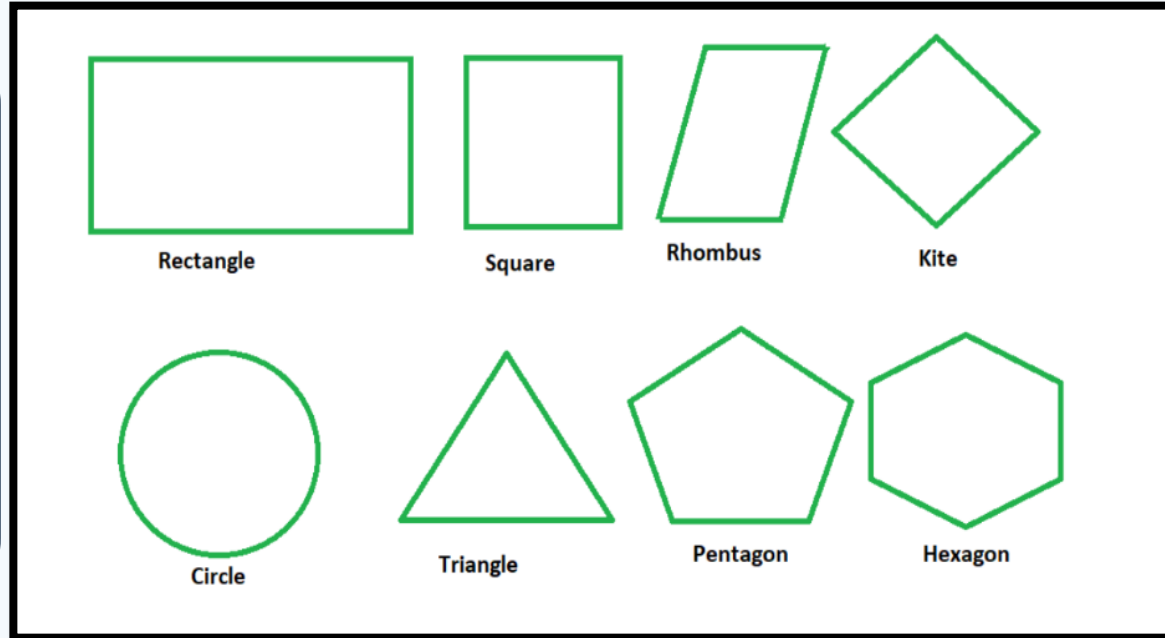




Introduce to Multiplication

Practice: Perimeter of each polygon

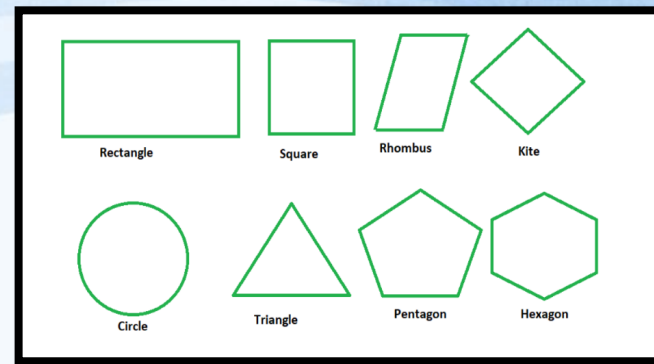
1. Rectangle
2. Square
3. Rhombus
4. Kite
5. Circle
6. Equal Triangle
7. Regular Pentagon
8. Regular Heptagon(7)
9. Regular Hexagon





Introduce to Multiplication

Practice: What is Perimeter?



1. Rectangle: $L=8, W=3$

2. Square: $S=6$

3. Rhombus: $L=5, W=6$

4. Kite: $S=10$

5. Circle: $R=3$

6. Equal Triangle: $S=4$

7. Regular Pentagon: $S=5$

8. Regular Heptagon(7): $S=9$

9. Regular Hexagon: $S=10$

1. $P= 2 \times 8 + 2 \times 3 = 22$

2. $P= 4 \times 6 = 24$

3. $P = 2 \times 5 + 2 \times 6 = 22$

4. $P=4 \times 10 = 40$

5. $P=2 \pi R$

6. $P=3 \times 4 = 12$

7. $P=5 \times 5 = 25$

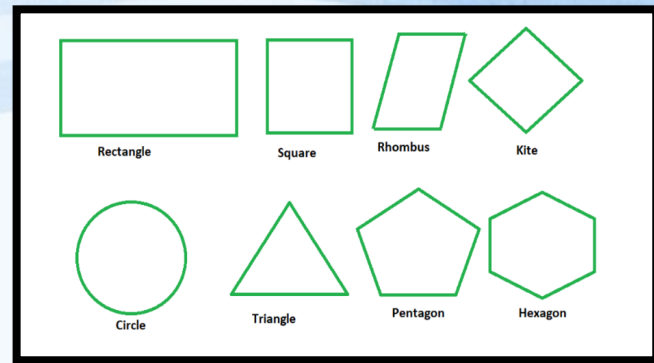
8. $P=9 \times 7 = 63$

9. $P=10 \times 8 = 80$



Introduce to Multiplication

Practice: 1 What is Area?



1. Rectangle: $L=8$, $W=3$

$$1. A = L \times W = 24$$

2. Square: $S=6$

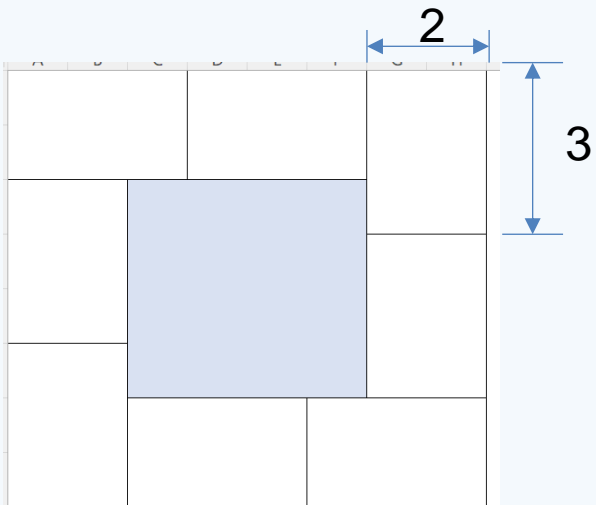
$$2. A = S \times S = 6 \times 6 = 36$$



Introduce to Multiplication

Practice 7

Eight white 2 by 3 rectangles are arranged as show. What is the area of the shaded square they surround?



Answer: 16

$$(3+3+2) \times (3+3+2) - 2 \times 3 \times 8 = 16$$

$$\text{Or } (3+1) \times (3+1) = 16$$

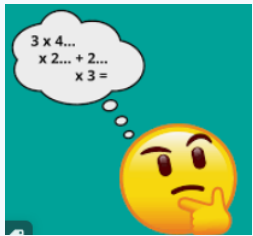


Introduce to Multiplication

Practice 7

Lizzies adds two whole numbers and gets a sum of 12. What is the greatest possible product of Lizzie's two numbers?

$$12 = ? + ?$$



Possible: addend of 12:

1 by 11: (or 11 by 1) Sum=12, $M=1 \times 11 = 11$

2 by 10: (or 10 by 2) Sum=12, $M=2 \times 10 = 20$

3 by 9: (or 9 by 3) Sum = 12, $M=3 \times 9=27$

4 by 8: (or 8 by 4) Sum=12, $M=4 \times 8 = 32$

5 by 7: (or 7 by 5) Sum=12, $M=5 \times 7 = 35$

6 by 6: (or 6 by 6) Sum=12, $M=6 \times 6 = 36$



Introduce to Multiplication

Practice 2:

Grogg buys some gumballs from the candy store. Walking back from the store, he loses half his gumballs. Then, he gives half of the remaining gumballs to Alex. Then Grogg chews half of the gumballs he has left. At the end, he has 11 gumballs. How many gumballs did Grogg original buy from the candy store?

Answer: $2 * 2 * (11 \times 2) = 88$ gumballs



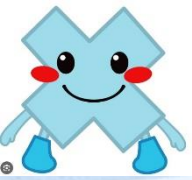
Introduce to Multiplication

Practice 4:

There are fewer than 75 students in the Beast Academy Marching Band. When they stand in rows of 3, there is 1 extra band member. When they stand in rows of 4, there are 2 extra band members. When they stand in rows of 5, there are 3 extra band members. How many students are in the Beast Academy Marching Band?

Answer: 58

$$58/3=19+R1, \quad 58/4=14+R2, \quad 58/5 = 11+R3$$



Introduce to Multiplication

Practice 7

$$15 \times 15 = 10 \times 20 + 5 \times 5 = 200 + 25 = 225$$

5			
10			
	10	5	

			5	
				5
10				
	10	5	5	



Introduce to Multiplication

Practice 12

$$A5 * A5 = A0 * (A+1)0 + 5 \times 5 = A*(A+1) + 25$$

$$A5 * A5 = \underline{A*(A+1)} \underline{25}$$



Introduce to Multiplication

Practice 11

$$75 \times 75 = 70 \times 80 + 5 \times 5 = 5600 + 25 = 5625$$

$$75 \times 75 = (7 \times 8) (5 \times 5) = 5625$$

7+1

$$85 \times 85 = 80 \times 90 + 5 \times 5 = 7200 + 25 = 7225$$

$$85 \times 85 = (8 \times 9) (5 \times 5) = 72 25$$

$$95 \times 95 = 90 \times 100 + 5 \times 5 = 9000 + 25 = 9025$$

$$95 \times 95 = (9 \times 10) (5 \times 5) = 90 25$$



Introduce to Multiplication

Practice 12

$$AB \times AB = (A(B-1) \times A(B-1)) + (A(B-1) + AB)$$

$$12 \times 12 = (11 \times 11) + (11 + 12) = 144$$

$$13 \times 13 = (12 \times 12) + (12 + 13) = 144 + 25 = 169$$

$$14 \times 14 = (13 \times 13) + (13 + 14) = 169 + 27 = 196$$



Introduce to Multiplication

Practice 18

$$200 \times 200 =$$

$$201 \times 201 =$$

$$202 \times 202 =$$

$$200 \times 200 = 40000$$

$$\begin{aligned} 201 \times 201 &= 200 \times 200 + (200 + 201) \\ &= 40000 + 401 = 40401 \end{aligned}$$

$$\begin{aligned} 202 \times 202 &= 201 \times 201 + (201 + 202) \\ &= 40401 + 403 = 40804 \end{aligned}$$



Introduce to Multiplication

Practice 3



How many Ostriches?

Sophia is excited to visit her grandfather's farm. In the field, Sophia sees lanky ostriches and cows. She cannot help counting them. If Sophia counts a total of 30 heads, and 86 legs for these ostriches and cows, how many ostriches are there?

Answer: **17**

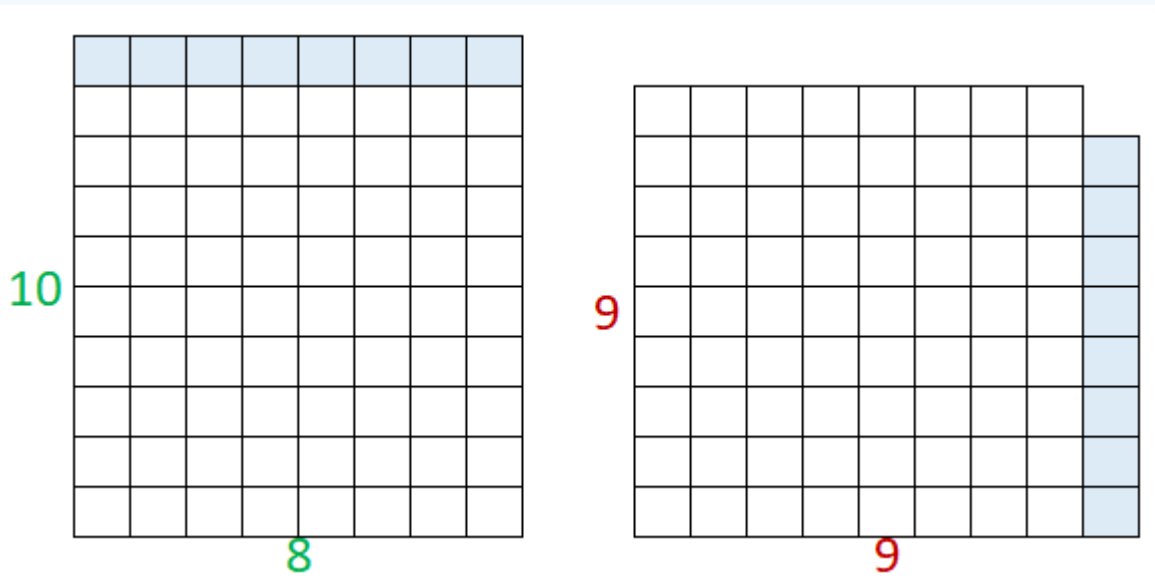


Introduce to Multiplication

Practice 25

Multiply numbers that are exactly two apart (like 8 and 10)

$$8 \times 10 = ?$$



$$\begin{aligned} 8 \times 10 &= 9 \times 9 - 1 \\ &= 81 - 1 = 80 \end{aligned}$$



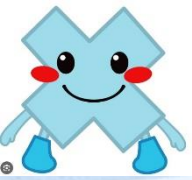
Introduce to Multiplication

Practice 26

$$9 \times 11 = ? \quad = 10 \times 10 - 1 = 100 - 1 = 99$$

$$14 \times 16 = \quad 15 \times 15 - 1 = 225 - 1 = 224$$

$$19 \times 21 = \quad 20 \times 20 - 1 = 400 - 1 = 399$$



Introduce to Multiplication

Practice 29

$$\begin{aligned} 3001 \times 2999 = ? &= 3000 \times 3000 - 1 \\ &= 9\,000,000 - 1 = 8,999,999 \end{aligned}$$

$$\begin{aligned} 8999 \times 9,001 &= 9000 \times 9000 - 1 \\ &= 81\,000,000 - 1 = 80,999,999 \end{aligned}$$

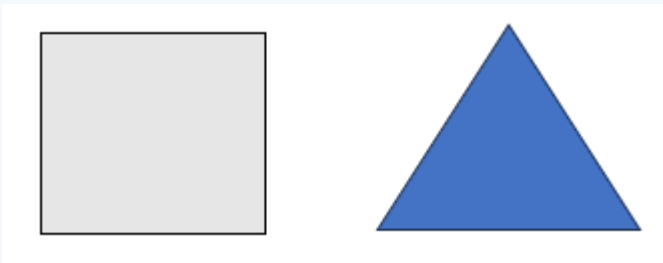
$$104 \times 106 = 105 \times 105 - 1 = 11,025 - 1 = 11,024$$



Word problems

Practice 126 (p70)

The square and the equilateral triangle below have the same perimeter. The side length of the triangle is 8. What is the area of the square?



$$\begin{aligned} \text{Triangle Perimeter:} \\ = 8 \times 3 = 24 \end{aligned}$$

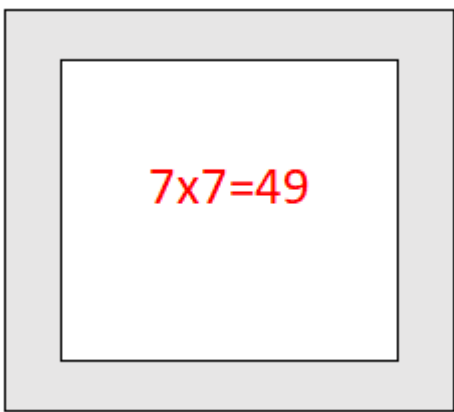
$$\begin{aligned} \text{Side of square: } 24 / 4 &= 6 \\ \text{Area of square} &= 6 \times 6 = 36 \end{aligned}$$



Word problems

Practice 129 (p71)

Lizzie has a square sheet of gray paper. Grogg cuts a 7 by 7 square from Lizzie's paper. The area of the remaining gray paper, shown below, is 32 squares. What is the side length of Lizzie's original square paper?



Grogg cut: $7 \times 7 = 49$

Gray area: 32

Total area: $49 + 32 = 81$

Side of original square: 9